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Hydrodynamic model of unsteady movement of oil and gas in the reservoir-well system

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Abstract. A hydrodynamic model of unsteady movement of oil and gas in the reservoir-well system is constructed and solutions to the system of differential equations are given. The obtained analytical formula makes it possible to determine the dynamics of the pressure at the bottom of the well and the productivity of the formation, depending on the parameters of the system. Numerical calculations have been carried out for practical values of the reservoir-well system parameters.

Keywords: reservoir-wells, Laplace transform, fluid movement, gas movement, continuity equation.

Introduction

Gas lifting of liquid from a well is widely used in oil production. The increase in the efficiency of the hoist is of great practical and scientific importance. Despite the fact that a lot of works [1-6] are devoted to this problem, the issue of increasing the efficiency of gas lift taking into account the movement of fluid in the reservoir-well system is still poorly understood. And the movement of fluid during oil production occurs in the reservoir-well system. Therefore, modeling and studying the movement of gas lift fluid in the reservoir-well system is of great practical and scientific importance.

Statement and solution of the problem

Consider plane-radial filtration of a homogeneous fluid in a circular homogeneous reservoir.

The differential equation of the piezoconductivity of a plane-radial fluid flow has the form [7,8]

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\partial \Delta P}{\partial r}\right)=\frac{1}{\chi}\frac{\partial \Delta P}{\partial t} \qquad r_c \le r \le R_k;\ t>0. \tag{1}$$

where $\Delta P = P_k - P$; $\chi = \dfrac{k}{\mu\beta^*}$.

Initial and boundary conditions

$$\Delta P\Big|_{t=0} = \frac{P_k - P_c(0)}{\ln\dfrac{R_k}{r_c}} \ln\frac{R_k}{r}, \qquad r_c \le r \le R_k; \tag{2}$$

$$\Delta P\Big|_{r=R_k} = 0, \qquad t > 0; \tag{3}$$

$$\Delta P\Big|_{r=r_c} = P_k - P_c(0), \qquad t > 0 \tag{4}$$

The solution of equation (1) with the initial (2) and boundary conditions (3) and (4) has the form [7,9]

$$\Delta P(r,t) = (P_k - P_c(0))\left[\frac{\ln\left(\dfrac{R_k}{r}\right)}{\ln\left(\dfrac{R_k}{r_c}\right)}\cdot\frac{\Delta P_{cy}}{\Delta P_{c1}} - \pi\sum_{i=1}^{\infty} A(x_\nu\frac{R_k}{r_c})u\left(x_\nu\frac{r}{r_c}\right)\cdot\exp\left(-\frac{x_\nu^2\,\chi t}{r_c^2}\right)\right] -$$
$$-\int_0^t \dot{P}_c(\tau)\pi\left[\frac{\ln\left(\dfrac{R_k}{r}\right)}{\ln\left(\dfrac{R_k}{r_c}\right)}\cdot\frac{\Delta P_{cy}}{\Delta P_{c1}} - \pi\sum_{i=1}^{\infty} A(x_\nu\frac{R_k}{r_c})u\left(x_\nu\frac{r}{r_c}\right)\cdot\exp\left(-\frac{x_\nu^2\,\chi(t-\tau)}{r_c^2}\right)\right]d\tau \tag{5}$$

where $\Delta P_{cy} = P_k - P_c(0)$, $P_c(0)$ pressure on the borehole wall, at the initial moment of time, $\Delta P_{c1} = P_c(0) - P_{c1}$ differential pressure to which the initial differential ΔP_{cy} decreases, P_{c1} - pressure on the borehole wall at the bottom of the borehole after pressure change $P_c(0)$

$$u_\nu\left(\frac{x_\nu r}{r_c}\right) = J_0\left(\frac{x_\nu r}{r_c}\right)Y_0\left(\frac{x_\nu R_k}{r_c}\right) - Y_0\left(\frac{x_\nu r}{r_c}\right)J_0\left(\frac{x_\nu R_k}{r_c}\right)$$

x_ν - roots of the transcendental equation

$$J_0\left(\frac{x R_k}{r_c}\right)Y_0(x) - J_0(x)Y_0\left(\frac{x R_k}{r_c}\right) = 0 \tag{6}$$

$$A_\nu\left(x_\nu\frac{R_k}{r_c}\right) = \frac{J_0(x_\nu\dfrac{R_k}{r_c})J_0(x_\nu)}{J_0^2(x_\nu\dfrac{R_k}{r_c}) - J_0^2(x_\nu)}$$

The fluid flow rate at the moment t through the lateral surface of the well with radius is determined by the formula

$$Q\Big|_{r=r_c} = -2\pi r_c h \frac{k}{\mu}\frac{\partial \Delta P}{\partial r}\Big|_{r=r_c} \tag{7}$$

Then from expression (7) taking into account formula (5), we obtain

$$Q_{смес}(t) = 2\pi h\frac{k}{\mu}(P_k - P_c(0))\left(\frac{1}{\ln\left(\frac{R_k}{r_c}\right)}\frac{\Delta P_{cy}}{\Delta P_{c_1}} - 2B_\nu\left(x_\nu\frac{R_k}{r_c}\right)\cdot\exp\left(-b_\nu t\right)\right) -$$

$$- 2\pi h\frac{k}{\mu}\int_0^t \dot{P}_c(\tau)\left(\frac{1}{\ln\left(\frac{R_k}{r_c}\right)}\frac{\Delta P_{cy}}{\Delta P_{c_1}} - 2B_\nu\left(x_\nu\frac{R_k}{r_c}\right)\exp\left(-b_\nu(t-\tau)\right)\right)d\tau. \tag{8}$$

where $$B_\nu\left(x_\nu\frac{R_k}{r_c}\right) = \frac{J_0^2\left(x_\nu\frac{R_k}{r_c}\right)}{J_0^2(x_\nu) - J_0^2\left(x_\nu\frac{R_k}{r_c}\right)}, b_\nu = \left(\frac{x_\nu^2\chi}{r_c^2}\right)$$

Gas movement in annular space.

Now consider the motion of gas in the annular space. The equations of gas motion in annular space and continuity are described by the equations of I.A. Charny [10,11]

$$-\frac{\partial P}{\partial x} = \frac{\partial Q}{\partial t} + 2aQ, \qquad -\frac{\partial P}{\partial t} = c^2\frac{\partial Q}{\partial x}, \quad Q = \rho u \tag{9}$$

Differentiating the first equation of expression (9) in the x, coordinate, and the second in time t, and subtract one from the other and get:

$$\frac{\partial^2 P}{\partial t^2} = c^2\frac{\partial^2 P}{\partial x^2} - 2a\frac{\partial P}{\partial t} \tag{10}$$

Having placed the origin of the coordinate axis at the wellhead and directed it downward, for the initial and boundary conditions we will have:

$$P\Big|_{t=0} = 0, \quad 0 \le x \le l; \tag{11}$$

$$\frac{\partial P}{\partial t}\Big|_{t=0} = 0, \quad 0 \le x \le l; \tag{12}$$

$$P\Big|_{x=0} = P_0(t), \quad t > 0; \tag{13}$$

$$P\big|_{x=l} = P_c(0), \quad t > 0. \tag{14}$$

where $P_0(t)$ - gas injection pressure, $P_c(t)$ - bottomhole pressure, c -speed of sound in gas, a - drag coefficient, t time x-coordinate, $Q = \rho u$ - mass flow rate of gas in the flow area of the annular space, ρ - gas density at a given pressure, P-pressure in any cross-section of the annular space.

The solution of equation (10), taking into account the boundary conditions (13) and (14), will be sought in the form [12,13]

$$P = P_0(t) - \frac{P_0(t) - P_c(t)}{l} x + \sum_{i=1}^{\infty} \varphi_i \sin\frac{i\pi x}{l} \tag{15}$$

Substituting expression (15) into equation (10), multiplying both sides of the resulting expression by $\sin\frac{i\pi x}{l}$ and integrating it from 0 to l, we get the equation:

$$\begin{aligned}&\ddot{\varphi}_i + 2a\dot{\varphi}_i + c^2\frac{i^2\pi^2}{l^2}\varphi_i = -\frac{4a\dot{P}_0(t)}{i\pi} + \frac{4a}{i\pi}(-1)^i \dot{P}_c(t) - \\ &- \ddot{P}_0(t)\frac{2}{i\pi} + \frac{2}{i\pi}(-1)^i \ddot{P}_c(t)\end{aligned} \tag{16}$$

Applying the Laplace transform [14,15] from expression (16) we obtain

$$\begin{aligned}\overline{\varphi_i}(s) &= \frac{S}{(s+a)^2 + \omega_i^2}\varphi_i(0) + \frac{1}{(s+a)^2 + \omega_i^2}\dot{\varphi}_i(0) + \frac{1}{(s+a)^2 + \omega_i^2}2a\varphi_i(0) + \\ &+ \frac{1}{(s+a)^2 + \omega_i^2}\frac{2}{i\pi}(-1)^i \overline{\ddot{P}}_c + \frac{1}{(s+a)^2 + \omega_i^2}\frac{4a}{i\pi}(-1)^i \overline{\dot{P}}_c - \\ &- \frac{1}{(s+a)^2 + \omega_i^2}\overline{\ddot{P}}_0\frac{2}{i\pi} - \frac{1}{(s+a)^2 + \omega_i^2}\frac{4a\overline{\dot{P}}_0}{i\pi}\end{aligned} \tag{17}$$

$\varphi_i(0)$ and $\dot{\varphi}_i(0)$ are determined from expression (15) and have the form

$$\varphi_i(0) = -\frac{2}{\pi}\left[P_0(0) + P_c(0)\right], \qquad \dot{\varphi}_i(0) = 0$$

где $P_0(0)$ *и* $P_c(0)$ respectively, the initial value of the gas supply pressure $P_0(t)$ and bottomhole pressure $P_c(t)$.

Applying the Laplace transform from expression (15), we obtain

$$\overline{P} = \overline{P_0}(t) - \frac{\overline{P}_0 - \overline{P}_c}{l}x + \sum_{i=1}^{\infty}\overline{\varphi}_i(t)\sin\frac{i\pi x}{l} \tag{18}$$

Substituting expression (17) into formula (18), and the resulting expression into the first equation of system (9) after the Laplace transform, we obtain:

$$\overline{Q}_{газ} = \frac{\overline{P}_0}{l(s+2a)} - \frac{\overline{P}_c}{l(s+2a)} + \sum_{i=1}^{\infty} \frac{\overline{\varphi}_i}{(s+2a)} \cos\frac{i\pi x}{l} + \frac{Q_{газ}(0)}{(s+2a)} \tag{19}$$

where $Q_{газ}(0)$ - initial mass flow rate of gas.

The movement of the gas-liquid mixture in the riser pipe.

The gas supplied through the annular space and the liquid coming from the formation are mixed in the riser string and rises through it. The density of the mixture in this case can be determined by the formula [6]

$$\frac{1+\eta}{\rho_{см}} = \frac{1}{\rho_г} + \frac{\eta}{\rho_н}, \qquad \rho_{см} = \frac{(1+\eta)\rho_н \rho_г}{\rho_н + \eta\rho_г}$$

where $\rho_{см}$, $\rho_н$, $\rho_г$ - respectively, the density of the mixture, oil and gas at a given pressure, η - mass fraction of oil in gas.

In the first approximation, the mixture is assumed to be homogeneous and the interactions between the liquid and gas bubbles are neglected.

Then the equation of motion of the mixture will have the same form as equation (10). By placing the origin of the x_1 coordinate axis at the lower end of the pipe string and directing it upwards for the initial and boundary conditions, we will have

$$P\big|_{t=0} = 0, \quad 0 \le x_1 \le l \tag{20}$$

$$\left.\frac{\partial P}{\partial t}\right|_{t=0} = 0, \quad 0 \le x_1 \le l \tag{21}$$

$$P\big|_{x1=0} = P_c(t), \quad t > 0 \tag{22}$$

$$P\big|_{x1=l} = P_{уст}(t), \quad t > 0 \tag{23}$$

The solution of equation (10), taking into account the boundary conditions (22) and (23), will be sought in the form [9]

$$P = P_c(t) - \frac{P_c - P_{уст}}{l} x_1 + \sum_{i=1}^{\infty} \varphi_{1i} \sin\frac{i\pi x_1}{l} \tag{24}$$

Substituting expression (24) into equation (10), multiplying both sides of the resulting expression by $\sin\frac{i\pi x_1}{l}$ and integrating it from 0 to l, we will have:

$$\ddot{\varphi}_{1i}+2a\dot{\varphi}_{1i}+c^2\frac{i^2\pi^2}{l^2}\varphi_{1i}=-\frac{4a\dot{P}_c}{i\pi}+\frac{4a}{i\pi}(-1)^i\dot{P}_{yct}-$$
$$-\ddot{P}_c\frac{2}{i\pi}+\frac{2}{i\pi}(-1)^i\ddot{P}_{yct} \quad (25)$$

Applying the Laplace transform from expression (24), we obtain

$$\overline{\varphi_{1i}}(s)=\frac{S}{(s+a)^2+\omega_i^2}\varphi_{1i}(0)+\frac{1}{(s+a)^2+\omega_i^2}\dot{\varphi}_{1i}(0)+\frac{1}{(s+a)^2+\omega_i^2}2a\varphi_{1i}(0)+$$
$$+\frac{1}{(s+a)^2+\omega_i^2}\frac{2}{i\pi}(-1)^i\overline{\ddot{P}}_{yct}+\frac{1}{(s+a)^2+\omega_i^2}\frac{4a}{i\pi}(-1)^i\overline{\dot{P}}_{yct}-$$
$$-\frac{1}{(s+a)^2+\omega_i^2}\overline{\ddot{P}}_c\frac{2}{i\pi}-\frac{1}{(s+a)^2+\omega_i^2}\frac{4a\overline{\dot{P}}_c}{i\pi} \quad (26)$$

$\varphi_{1i}(0)$ and $\dot{\varphi}_{1i}(0)$ are determined from expression (24) taking into account the initial conditions (20) and (21) and have the form

$$\varphi_{1i}(0)=-\frac{2}{\pi}\left[P_c(0)+P_{ycm}(0)\right], \qquad \dot{\varphi}_{1i}(0)=0$$

where $c^2\frac{i^2\pi^2}{l^2}-a^2=\omega_i^2$

Applying the Laplace transform from expression (24), we obtain

$$\overline{P}=\overline{P}_c(t)-\frac{\overline{P}_c-\overline{P}_{yct}}{l}x_1+\sum_{i=1}^{\infty}\overline{\varphi}_{1i}\sin\frac{i\pi x_1}{l} \quad (27)$$

Substituting formula (27) into the first equation, system (9) after the Laplace transform, we obtain

$$\overline{Q}_{smes}=\frac{\overline{P}_c}{l(s+2a)}-\frac{\overline{P}_{yct}}{l(s+2a)}-\sum_{i=1}^{\infty}\frac{\overline{\varphi}_{1i}}{(s+2a)}\cdot\frac{i\pi}{l}\cos\frac{i\pi x_1}{l}+\frac{Q_{smes}(0)}{(s+2a)} \quad (28)$$

where $Q_{smes}(0)$ -the initial value of the mass flow rate of the mixture.

Continuity conditions

$$f_k Q_{qaz}\big|_{x=l}+Q_{\phi}\big|_{r=r_c}=f_T Q_{smes}\big|_{x_1=0} \quad (29)$$

where f_T and f_k -respectively, the area of the flow section of the pipe string and the annular section. Applying the Laplace transform from expression (29), we obtain

$$f_k\overline{Q}_{qaz}\big|_{x=l}+\overline{Q}_{\phi}\big|_{r=r_c}=f_T\overline{Q}_{c\text{м}}\big|_{x_1=0} \quad (30)$$

Substituting expressions (8), (19), and (28) into the continuity condition (30), taking into account only one term of the series in the first approximation, we obtain

$$\overline{P}_c = 2\pi h\rho_{жс}\frac{k}{\mu}\frac{1}{\ln\left(\frac{R_k}{r_c}\right)}\cdot\frac{\Delta P_{cy}^2}{\Delta P_{c1}}(s+2a)(s+b_\nu)((s+a)^2+\omega_i^2)\cdot\frac{1}{s\psi(s)} -$$

$$-4\pi h\frac{k}{\mu}\rho_{жс}B_\nu\left(x_\nu\ \frac{r_c}{R_k}\right)\cdot\Delta P_{cy}\ \frac{(s+2a)((s+a)^2+\omega_i^2)}{\psi(s)} +$$

$$+2\pi h\rho_{жс}\frac{k}{\mu}P_c(0)\cdot\frac{1}{\ln\left(\frac{R_k}{r_c}\right)}\cdot\frac{\Delta P_{cy}}{\Delta P_{c1}}(s+2a)(s+b_\nu)((s+a)^2+\omega_i^2)\cdot\frac{1}{s\psi(s)} -$$

$$-4\pi h\frac{k}{\mu}\rho_{жс}B_\nu\left(x_\nu\ \frac{R_k}{r_c}\right)\cdot P_c(0)\frac{(s+2a)((s+a)^2+\omega_i^2)}{\psi(s)} +$$

$$+\frac{f_k}{l}\cdot(s+b_\nu)\cdot((s+a)^2+\omega_i^2)\cdot\frac{\overline{P_0}}{s\psi(s)}+\frac{f_k}{l}\cdot i\pi\cdot(s+b_\nu)\frac{1}{\psi(s)}\cdot\left[s\varphi_i(0)+\dot{\varphi}_i(0)+\right.$$

$$\left.+2a\varphi_i(0)-\frac{2}{\pi}(sP_c(0)+\dot{P}_c(0))+\frac{4}{\pi}P_c(0)-\frac{2}{\pi}\overline{\ddot{P}_0}-\frac{4a}{\pi}\overline{\dot{P}_0}\right]+$$

$$+\frac{f_k Q_q(0)(s+b_\nu)((s+a)^2+\omega_i^2)}{\psi(s)}+\frac{f_t\overline{P}_{уcт}(t)(s+b_\nu)((s+a)^2+\omega_i^2)}{l\cdot\psi(s)}+$$

$$+\frac{f_t\pi(s+b_\nu)}{l\cdot\psi(s)}\cdot\left[s\varphi_{i1}(0)+\dot{\varphi}_{1i}(0)+2a\varphi_{1i}(0)-\frac{2}{\pi}\overline{\ddot{P}}_{уcт}-\right.$$

$$\left.-\frac{4}{i\pi}\overline{\dot{P}}_{уcт}+\frac{2}{\pi}(sP_c(0)+\dot{P}_c(0)+\frac{4a}{\pi}P_c(0)\right]-\frac{f_t Q_{smes}(0)(s+b_\nu)((s+a)^2+\omega_i^2)}{\psi(s)} \qquad (31)$$

$$\psi(s)=F(s)\cdot\left[(s+a)^2+\omega_i^2\right]+2\frac{f_k}{l}(s+b_\nu)+4a\frac{f_k}{l}(s+b_\nu)s+$$

$$+2\frac{f_t}{l}(s+b_\nu)s^2+4a\frac{f_t}{l}(s+b_\nu)s$$

where ,

$$F(s)=2\pi h\rho_{жс}\frac{k}{\mu}\frac{1}{\ln\left(\frac{R_k}{r_c}\right)}\cdot\frac{\Delta P_{cy}}{\Delta P_{c1}}(s+2a)(s+b_\nu)-4\pi h\frac{k}{\mu}\rho_{жс}B_\nu\left(x_\nu\ \frac{R_k}{r_c}\right)\cdot s\cdot(s+2a)+$$

$$+\frac{f_k}{l}(s+b_\nu)+\frac{f_t}{l}(s+b_\nu)$$

$Q_{см}(0)$ and $Q_г(0)$ are determined from the following formulas

$$Q_{см}(0)=Q_г(0)+\frac{\rho_{жс}Q_ф(0)}{2\pi r_c h},\quad Q_ф(0)=2\pi h\frac{k}{\mu}\frac{P_k-P_c(0)}{\ln\frac{R_k}{r_c}}$$

$$Q_г(0)=\frac{P_0(0)\cdot\exp(g\frac{\rho_{ат}}{P_{ат}}l)-P_c(0)}{\exp(g\frac{\rho_{ат}}{P_{ат}}l)-1}\frac{\rho_{ат}g}{2aP_{ат}}$$

where ρ_{am}-gas density at atmospheric pressure; g-acceleration of gravity, P_{am}- atmospheric pressure; $Q_{ф}(0)$ - oil inflow from the reservoir per unit of time at the initial moment of time.

From expression (28), taking into account expressions (26) and (27), one can determine $Q_{см}(t)$.

Conclusion

A hydrodynamic model of unsteady movement of oil and gas in the reservoir-skavazhin system has been built and analytical expressions have been obtained that allow, depending on the parameters of the reservoir-well system, to determine the bottomhole pressure and well productivity.

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