Key positions of the implementation of continuity in the study of natural numbers and fractions at the initial and basic stages of education

Trofimenko Yulia Vladimirovna
Candidate of Pedagogic Sciences, Associate Professor
Krivets Anastasia Alexandrovna
undergraduate
Anton Chekhov Taganrog State Institute (branch) –

Rostov State University of Economics (RSUE)

Abstract. The article examines the topical issue at this stage of the implementation of the principle of continuity at different stages of mathematical education in the study of natural numbers and ordinary fractions. The authors put forward as key positions the scientific approach in the presentation of the indicated issues, the formation of a theoretical type of thinking, an orientation towards the developing principles of teaching mathematics.

Keywords. Mathematical education, continuity, scientific character, theoretical thinking, natural numbers.

The implementation of the idea of continuity at different levels of mathematics education is today an urgent problem, the solution of which is aimed at the research of many teachers, mathematicians, methodologists. The modernization of the school mathematics course meets the modern requirements of the development of science, the introduction of mathematics in the most diverse areas of knowledge. At present, not only in our country, but also in many countries of the world, intensive searches are being carried out for ways to improve school teaching in mathematics, which would bring it closer to the modern level of development of mathematical science. The task of such teaching mathematics is:

– not the transfer of ready-made knowledge, but in the organization of students' activities to master the system of knowledge, methods of analysis and generalization of educational material based on practical experience, knowledge gained during the period of informal learning (from everyday life, communication with adults, programs of additional educational institutions);

– systematic assimilation of knowledge, skills, skills, subject competences for the use of mathematical representations and concepts in the process of presenting information about the features of surrounding objects, the course of development of any phenomenon in quantitative and spatial terms;

– creative use of the acquired knowledge in solving theoretical and practical problems;

– "the formation of the ability for continuous mental activity, the foundations of logical thinking, spatial imagination, mathematical speech and argumentation, the ability to distinguish between justified and unfounded judgments" [1, p. 28].

In accordance with the Model Program in Mathematics that meets the requirements of the new standards, you attach particular importance to mathematics as a subject in the mental development of the subject of study. The purpose of mathematical development is "the formation of the ability for intellectual activity (logical and sign-symbolic thinking), spatial imagination, mathematical speech; the ability to build reasoning, choose argumentation, search for information; understand the meaning of quantities and how to measure them; the development of interest in mathematics, the desire to use mathematical knowledge in everyday life"[6].

The solution of the indicated tasks and goals of teaching mathematics lies in:

– difficulties associated with the specifics of the subject of mathematics, its logical and psychological foundations of design, which requires the cognitive side of education, which is a powerful means of developing younger schoolchildren and a strong, faithful internal support of the present, original, not copied mastery of knowledge, skills, and abilities;

– on the peculiarities of perception, thinking, memory of schoolchildren (not only their acquisition of knowledge, but also the mastery of techniques and methods of obtaining them, requiring mental effort and adequate application);

– on the diversity of the current state of pedagogical science and practice, characterized by the variability of concepts, technologies, methodological and methodological systems and approaches;

– implementation of the idea of continuity at different levels of mathematics education.

How can a teacher understand the merits and demerits of this or that, often compulsory for the leadership of an educational institution, training program, how to learn to consider all issues of the program of mathematical education in interrelation and continuity?

Of course, first of all, we are witnessing a new stage in the development of mathematical education, which is determined by the rejection of a uniform, unitary system of schooling. However, already between elementary and basic school, new contradictions arise more and more often, connected with the problem of continuity and continuity in teaching mathematics. The essence of these contradictions is as follows. As you know, fundamental scientific research on the problem of the relationship between learning and development was carried out in dignitaries on younger students. This led to the fact that the primary school has a fundamental psychological and pedagogical base for the implementation of the ideas of developing education at the methodological level. However, these ideas have not yet received proper development in the mathematics course of 5-6 classes. Secondary school methodologists see the solution to the problem of continuity between primary and secondary schools in the creation of a unified concept of mathematical education, the main principle of which is the priority of the developmental function, based on the scientific presentation of all formed mathematical concepts from the initial stages of education. However, declaring the priority of the developmental function of teaching, one must not forget that it is in the primary grades that the ability to learn should be formed, on the basis of which all subsequent mathematical education will be built.

Thus, the Concept for the Development of Mathematical Education in the Russian Federation states that "the initial stage of teaching mathematics has two main goals: internal (didactic) - preparing students for continuing education - and external (pragmatic) - the formation of mathematical literacy" [5]. Some authors, highlighting the external goal as a priority, argue that the content of elementary mathematics education should be "relatively closed" [2, p. 31]. In other words, the pragmatic goal - the formation of mathematical literacy - corresponds to the thesis about the sufficiency of primary education for a person's everyday life.

In this concept, teaching in grades 5-6 is also assigned a preparatory role, where it is already a question of the functional literacy of students. The closest to this concept is the statement of M.V. Lomonosov that "mathematics should be studied for that reason, that it puts the mind in order."

Summarizing the experience of teachers, as well as their own experience in mathematical training at different levels of education, we note that in order to characterize the concept of continuity in the framework of mathematical education, training must be considered as a process of the formation of a person's personality through mastering the basics of mathematical knowledge, skills, experience of mathematical activity. To characterize the continuity in teaching, an integrated, systematic approach is needed, which reflects: the logic of constructing the main content and methodological lines of the course, taking into account the relationship and development of the concepts studied by schoolchildren.

We will offer key positions for the implementation of continuity in the study of natural numbers and fractions at the initial and basic stages of training. In modern programs and textbooks, the method of studying natural numbers and fractions is carried out within the framework of the concept presented by N.B. Istomina [4], the main goal of which is to develop students' thinking in the process of assimilating mathematical content. The fundamental key of this concept is also in the study of natural numbers and fractions in grade 5, as well as rational numbers in grade 6. Thus, the main directions of the methodology for studying natural numbers in primary grades received their further development in the study of natural numbers and fractions in grade 5.

Let us characterize the teacher's activities aimed at the implementation of continuity in the mathematical education of schoolchildren.

An elementary school teacher, realizing that the cognitive development of a junior schoolchild takes place in the joint educational activity of him and his students, should be guided not by the type of thinking already mastered by the child, but by the new emerging type - by conceptual, theoretical thinking, because only in this case, learning will be truly developmental.

The traditional elementary course of mathematics with all its content is aimed at the intellectual development of younger students and has great opportunities for the successful organization of developmental education. Like most other initial courses in mathematics, it is largely based on the use of a number system built on the basis of the set-theoretic approach [3, p. 59]. Therefore, the teacher, in order to focus on the formation of theoretical thinking in junior schoolchildren in the process of teaching the elementary course of mathematics, it is necessary to know well the theoretical foundations of this number system, to see and use the possibilities of their application in the further educational process.

There are two independent approaches to constructing the set of non-negative integers - axiomatic and set-theoretic.

In a high school mathematics course, with the axiomatic construction of a set of non-negative integers, the concept of relations "more", "less" is introduced on the basis of the addition operation, already well studied by this time [2, p. 81]. As you know, in the traditional initial course of mathematics, this relation is considered earlier than addition. Therefore, consider the second approach to constructing a set of non-negative integers.

In the set-theoretic construction of the set of non-negative integers, the concepts "more", "less" are defined on the basis of a comparison of the sets representing the numbers under consideration. If these sets are not of equal cardinality, then the numbers under consideration are not equal, since they define different classes of equipotent finite sets. This requires the introduction of new concepts of comparison on the set of non-negative integers - the concepts of "more", "less" [4, p. 81].

Let us give a brief statement of the quantitative theory of non-negative integers, on the set-theoretical basis of its construction, which is the theoretical basis of the relations "more", "less".

Definition. A correspondence between elements of non-empty sets X and Y is a subset of the Cartesian product of the sets X and Y.

Definition. Let X and Y be non-empty sets. A bijection from a set X to a set Y is a correspondence f between elements of the sets X and Y, which has the following properties: a) to each element of the set X there corresponds some element of the set Y; b) different elements of the set X correspond to different elements of the set Y; c) each element of the set Y corresponds to some element of the set X; d) different elements of the set Y correspond to different elements of the set X. Designation: $f: X \leftrightarrow Y$.

Definition. Non-empty sets X and Y are called equipotent if there is a bijection from the set X to the set Y. Notation: $|X| = |Y|$.

Definition. Let X and Y be finite sets. There are as many elements in the set X as in the set Y, if X is equal to Y. The set X has fewer elements than in the set Y (or in the set Y there are more elements than in the set X), if X is equivalent to some sub-set of the set Y, provided that this subset is not equal to the set Y itself.

If on the elements of the set of finite sets M we consider the relation of equal cardinality ~ (tilde): $\sim \subset M \times M$ and ~ = {(*x*, *y*)| |*x*| = |*y*|}, and X, then, being an equivalence, it splits the set M into classes of equally powerful sets. Each class is assigned a name and a symbol to record it. For example, a class containing an empty set is assigned the name "zero" and the character to write it "0"; the class of equally powerful sets containing a set consisting only of the element d - the name "one" and the symbol 1, and so on.

Definition. Let M be a set whose elements are finite sets. On M, the ratio of equal power "set X is equal to set Y" is given.

A non-negative integer is the name of the class of the partition of the set M by the equivalence relation. A natural number is a non-zero non-negative integer.

From the last definition it immediately follows that $N_0 = N \cup \{0\}$ (N – is the designation of the set of natural numbers).

Definition. A representative of a non-negative integer p is a set that is an element of the class *p* of the partition of the set M by the equivalence relation ($M/_\sim$). Designation: capital letter of the Latin alphabet with subscript *p*.

Definition. A non-negative integer p is called the number of elements of the set X if the set X is a representative of the number p. Designation: *n*(X).

It's obvious, that $n(X) = p \Leftrightarrow X \in p$.

Definition. A non-negative integer *m* is less than a non-negative integer *p* (or a number *p* is greater than *m*) if there is a representative of the number *m*, which is a subset of some representative of the number *p* and is unequal to it. Designation: $m < p$ or $p > m$. $m < p \Leftrightarrow (\exists A_m, A_p)[A_m \subset A_p \wedge A_m \neq A_p]$, $p > m \Leftrightarrow (\exists A_m, A_p)[A_m \subset A_p \wedge A_m \neq A_p]$.

It is obvious from the definition, that $m < p \Leftrightarrow p > m$.

Definition. The set of all natural numbers ordered by the ratio "less" is called a natural series.

Designation: (N, <). For example, (N, <) = {1, 2, 3, 4, 5, 6, …}.

Definition. Let *p* be an arbitrary natural number. A *p*-segment of a natural number series is a set of all natural numbers, ordered by the relation "less than", each of which is less than or equal to *p*.

Designation: N_p. $N_p = \{1, 2, 3, 4, \ldots, p\}$.

For example: $N_1 = \{1\}$, $N_2 = \{1, 2\}$, $N_7 = \{1, 2, 3, 4, 5, 6, 7\}$.

Definition. A count (or numbering) of elements of a finite set A is a bijection of a set A onto some *p*-segment of a natural series; in this case, the image of an element is called its number.

Example. Count the elements of a set X = {*a, b, c, d, e*}.

Solution. Consider $f = \{(a, 1); (b, 2); (c, 3); (d, 4); (e, 5)\}$.

Since $f: X \leftrightarrow N_5$, then *f* – numbering of elements X by definition.

For convenience, it is customary to write the number with a subscript in the numbering. For example, $(d, 4) \Leftrightarrow d_4$ and $f = \{a_1, b_2, c_3, d_4, e_5\}$.

Note that the content of the elementary course in MEC "School of Russia" is based on the use of mainly a numerical system, in the construction of which a set-theoretic approach is used. Therefore, it is quite understandable that the method of studying the relationship "less", "more" on the set of non-negative integers in the first grade of traditional education uses a set-theoretic interpretation.

In accordance with this interpretation, the basis of the concept of relations "more", "less" on the set of non-negative integers is *the ratio of equal power of finite sets*. Let's remind him. One number is less than the other, if there is a representative of the first number, which is a subset of some representative of the second number, and the representatives of the numbers are unequal sets.

Preparatory work for the formation in students of the concept of the relationship "more", "less" (and at the same time the relationship "equal") on the basis of such an interpretation begins already in the pre-number period, at the very beginning of their study of the initial course of mathematics. This work is carried out on the basis of practical actions with various groups of objects. This approach makes it possible to use the experience previously accumulated by students and from the very beginning to teach in close connection with life and in the implementation of the principle of continuity.

Note that the ability to compare two numbers based on this method, that is, characterizing the comparison results with the words "more", "less", "equal", is one of *the main requirements for the mathematical training of students graduating from the first grade.* Using the example of studying the numbers of the first ten, younger students get acquainted with various ways of comparing numbers, one of which is based on comparing the corresponding groups of subjects. The main task of these lessons is "to teach junior schoolchildren to establish a mutually unambiguous correspondence between the objects of two sets". Its solution is closely related to the formation of an idea of quantity among junior schoolchildren. It is not excluded that the majority of students will turn to the counting of subjects, but the focus of children should be various methods of establishing a one-to-one correspondence [7, p. 19].

When organizing work on the formation of the concepts under consideration, subject, graphic and symbolic models are used. Therefore, in the process of this work, teachers' instructions for the most part will have to reflect precisely the stages of the modeling process and rely on the theoretical foundations of these concepts. Obviously, in this case, in order to formulate clear, concise and correct instructions, the teacher needs to know well the relevant theoretical material of the university course in mathematics.

Naturally, the instructions and instructions of the teacher for younger students cannot contain the terms of the university course in mathematics. But the teacher should be able to express their essence correctly in a language accessible to younger schoolchildren. He can do this only if he knows the theoretical basis of the concepts used well enough.

The basis of the concepts "more", "less" is the ratio of equal power of finite sets. And the concept "less" for non-negative integers can be expressed as follows: one number is less than the other, if in the set, the number of elements of which is equal to the first number, there are so many elements in the part of the set, the number of elements of which is equal to another number. For example, for a set A (a set of circles) and a set B (a set of rectangles), shown in Figure 1, it is true that sets A and B are not equal in power and, since from set B one can select a part in which there are the same number of rectangles, how many circles are in set A, then the number 4 (the number of circles) is less than the number 5 (the number of rectangles).

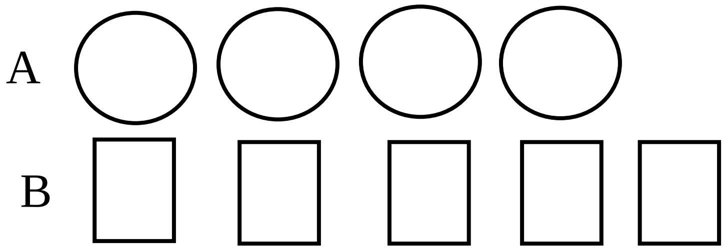

Fig. 1

Thus, the teacher must organize the educational process, the purpose of which is to form in younger schoolchildren the concepts of comparison relations, and hence the concepts "less", "more", so that younger students use a one-to-one correspondence between elements of sets or elements of one set and part of the elements of another set. Let us recall that they were trained to deliberately establish such a correspondence between subjects already in the pre-number period of studying the traditional elementary course in mathematics. Therefore, in the study of the relationship "more", "less" in the center of attention of students should be *various methods of establishing one-to-one correspondence, and not recounting the subjects of each group.* Establishment of a one-to-one correspondence between object aggregates is possible in various ways, the methodology of working with which the teacher must master.

The method of superimposing objects of one set on objects of another: consists in the fact that objects of one set are superimposed on objects of another. This method is used at the very beginning of acquainting students with the use of one-to-one correspondence between object populations to compare their numbers and requires direct action with objects.

But, since one of the goals of learning is to teach younger students to perform many actions, including the action of comparing the number, indirectly, in the mental plane, it is necessary to further use in learning methods that contribute to the advancement towards this goal. These methodists include the following methods.

The way of arranging objects of one set under objects of another set. This method is widely used at the stage of familiarizing younger students with the concepts of "less", "more", "equal" and requires direct action with objects. But since the one-to-one correspondence established with its help *must be recognized as a set of pairs*: an element of one set is an element of another set, then when it is completed, students develop imagination, which, undoubtedly, is an advance towards the specified goal.

For example, figure 2 is considered, the problem is posed: what is more (less) circles or squares?

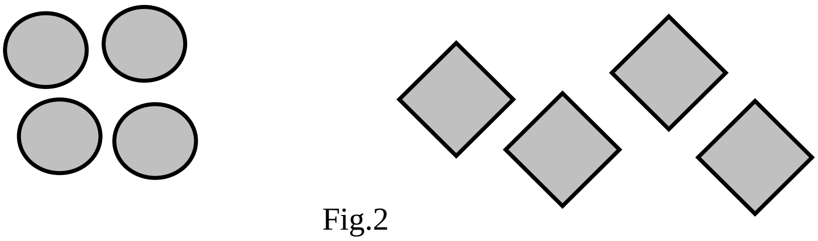

Fig.2

After completing the task based on the method of establishing correspondence between the sets of objects under consideration, figure 3 is obtained:

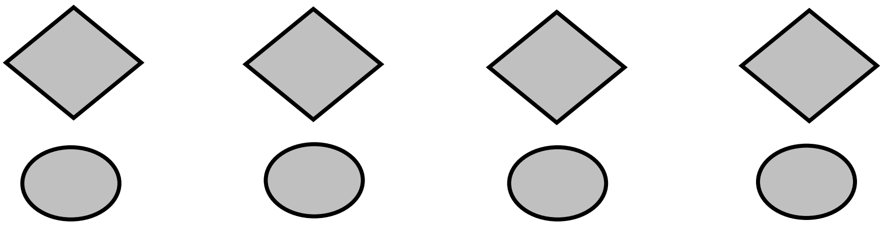

Fig. 3

And it is realized: *there are* **as many** *squares as there are circles.*

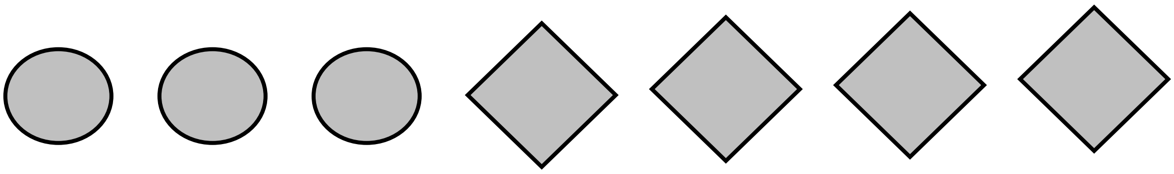

Fig. 4

If the sets shown in Figure 4 are considered, and a correspondence is established between them, then the correspondence model presented in Figure 5 is obtained, which helps students realize that *there are **more** squares than circles, and there are **fewer** circles than squares.*

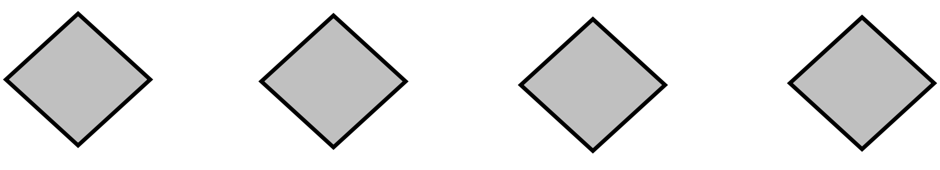

Fig. 5

The next method - **the method of forming pairs**, is the connection of each object of one set with each object of another, without placing the objects one under the other. This way of pairing is already based on the imagination of younger students. For example, Figures 6, 7.

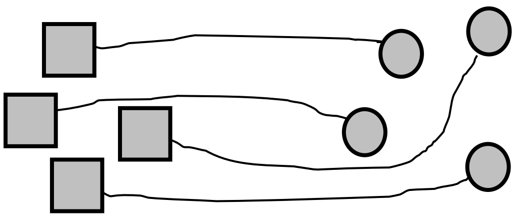

Fig. 6

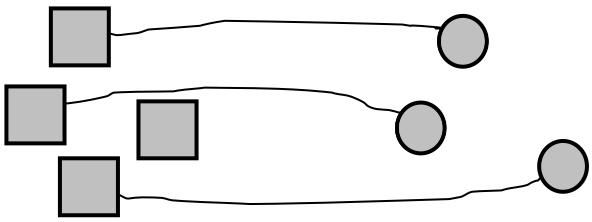

Fig. 7

The task of the teacher, when studying the concepts of the comparison relation, is to force younger students *not to use the recount of objects for the answer, but to concentrate their attention on establishing a one-to-one correspondence between the sets under consideration.*

Psychologists say that, taking into account the patterns of thinking in the learning process, when acquainting a student with any action that he needs to master, he (the student) is first introduced to the performance of actions on the corresponding material objects. And then, distracting from the properties of objects that are unnecessary in this case, they help to move from actions with material objects to action with their substitutes, free from all properties, except for those that are needed in this case. In other words, *go to the stage of materialized action*, which is some kind of graphic system, figurative or symbolic model*, on which, or with the help of which, the student performs the assimilated action.*

In order to carry out such a transition in the initial courses, a variety of tasks presented in the textbook are enough. In addition to the tasks of the textbook, the teacher can propose a system of specially designed tasks that allow you to move from the stage of materialized actions to a graphic or figurative model of establishing relations "less", "more" on the set of objects under consideration.

In building a methodological system for teaching the comparison of natural numbers in high school and further expanding the range of numbers under consideration, a high school teacher needs to build a teaching system so that students have the opportunity to independently acquire new knowledge on the basis of already formed methods of activity. Only in this case, the system of mathematical education will be of a developing nature, and, therefore, meet the requirements of the FSES and be based on the principle of continuity of all stages of mathematical education.

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