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Mathematical models of wave propagation on a soil layer with the property of non-linearly compressible and irreversible unloading with the base under the influence of a moving load[1]

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Abstract. The problem of the effect of a mobile load on a soil layer of finite thickness lying on a horizontal elastic foundation is considered.

The soil is modeled by an ideal nonlinearly compressible and irreversible unloading medium, in which the relationship between pressure and volumetric deformation under loading and during unloading of the medium is linear and irreversible.

The load is applied to the upper surface of the layer and moves at a superseismic speed. The problem of the effect of a moving load on a two-layer medium consisting of a soft soil layer and an elastic-yielding pad with different thicknesses and densities is considered. The solution to the problem is constructed analytically in both reverse and direct ways.

A two-layer medium consists of a soft soil layer of thickness *h* with an elastic deformable base. The soil is modeled by an inelastic ideal medium with linear compressibility and linear irreversible unloading. Consequently, the shear resistance of the medium is neglected. According

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to this statement, the influence of the deformability of the base and the load profile on the distribution of the dynamic parameters of the layer and the contact surface was investigated.

Keywords. Mathematical models, propagation, plastic wave, half-spaces, analytical solution, wave front, ideal fluid, linear compressibility, irreversible unloading, equation of motion, continuity, states of the medium.

Formulation of the problem. Let us consider the problem of the effect of a moving load on a two-layer medium consisting of a soft soil layer and an elastic-yielding pad with thicknesses h, h_1 and densities ρ, ρ_1. The soil is modeled by a nonlinearly compressible medium, and the pad, which has a weaker, than a soil with a stiffness K and a density $\rho_2 \leq \rho_1 -$ with a Winkler base. The lower boundary of the two-layer medium is solid and non-deformable. According to the accepted assumptions, the wave process in the spacer is neglected, and the compressed wave OA at $\xi \geq \xi_a$ from the contact surface of the two media is reflected in the form of the unloading wave AB of a strong rupture, and the behavior of the soil in regions 1, 2, 3, etc. is determined by the unloading branches of the $P \sim \varepsilon$ diagram.

The problem is of practical importance in assessing the levels of reduction of dynamic loads on various underground structures using a bulk screen with a resilient pad.

The solution to the problem is constructed analytically in both reverse and direct ways. Let's proceed with the presentation of these decisions. In the course of this task, the load profile $f(\xi)$, was determined, which in the future, when constructing solutions to the problem for areas 2 and 3, is considered given [1,2-5].

Taking into account that the medium in region 2 is in a state of unloading, then to solve the problem with respect to the velocity potential $\varphi_2(\xi,\eta)$ we have the equation

$$\mu^2 \frac{\partial^2 \varphi_2}{\partial \xi^2} - \frac{\partial^2 \varphi_2}{\partial \eta^2} = 0, \quad \left(\mu^2 = \frac{D^2}{C_p^2} - 1, \quad C_p = \sqrt{\frac{E}{\rho_1}} \right), \qquad (1)$$

with the following boundary conditions

$$tg\beta (V_1 - V_2) = (U_1 - U_2) \quad \text{at } \eta + \xi tg\beta = 2h_1, \qquad (2)$$

$$D \frac{\partial P_2}{\partial \xi} = K_x V_2, \quad \text{at } \eta = h, \quad \xi_a \leq \xi \leq \xi_c, \qquad (3)$$

Where $C_h = D\sin\beta, \quad tg\beta = 1/M, \quad K_x = K/h_2$, $U_2, V_2 -$ horizontal and vertical components of speed; $P_2 -$ medium pressure in area 2; $\beta -$ the angle of inclination of the reflected wave with the line AC; $K -$ Young's modulus spacers.

It is known that equation (1) for $D > C_p$ admits a solution of the form

$$\varphi_2(\xi,\eta) = f_1(\xi - \mu\eta) + f_2(\xi + \mu\eta). \quad (4)$$

Hence

$$U_2(\xi,\eta) = \frac{\partial\varphi_2}{\partial\xi} = f_1'(\xi - \mu\eta) + f_2'(\xi + \mu\eta),$$
$$V_2(\xi,\eta) = \frac{\partial\varphi_2}{\partial\xi} = -\mu f_1'(\xi - \mu\eta) + \mu f_2'(\xi + \mu\eta). \quad (5)$$

Substituting (5) into (2), after some transformations, we obtain

$$f_1'(z) = \frac{1}{2}\left[u_1\left(\frac{z+\mu(h+\xi_a tg\beta)}{2} - \frac{z-\mu(h+\xi_a tg\beta)}{2}\right) - \vartheta_1 tg\beta\left(\frac{z+\mu(h+\xi_a tg\beta)}{2} - \right.\right.$$
$$\left.\left. - \frac{z-\mu(h+\xi_a tg\beta)}{2}\right)\right]. \quad (5.6.6)$$

Substitute (5) into (3). Then we have

$$f_2'(z) + \frac{K_x\mu}{\rho_1 D^2} f_2'(z) = -f_1'(z - 2\mu h) + \frac{K_x\mu}{\rho_1 D^2} f_1'(z - 2\mu h), \quad (7)$$

where the dash above means the derivative with respect to the argument.

Equation (7) has a solution of the form

$$f_2'(z) = C_2 e^{-lz} - f_1'(z - 2\mu h) + f_1'(z - 2\mu h)e^{-l(z-z_0)} + 2le^{-lz}\int_{z_0}^{z} e^{lz} f_1'(z - 2\mu h)dz \text{ .(5.6.8),}$$

where $z_{02} = \xi_a \mu h, \quad l = (K_x\mu)/(\rho_1 d^2)$.

Obtain

$$C_2 = -e^{l(\xi_a+\mu h)}\left[U_1(\xi_a,h) - \frac{1}{\mu}V_1(\xi_a,h)_1\right]. \quad (9)$$

Thus, the solution to the problem in region 2 is expressed by the formulas

$$U_2(\xi,\eta) = \frac{1}{2}\Psi_{35}(\xi,\eta) - \frac{1}{2}\Psi_{36}(\xi,\eta) + le^{-l(\xi+\mu\eta)}\Psi_{37}(\xi,\eta), \quad (10)$$

$$V_2(\xi,\eta) = -\frac{\mu}{2}\Psi_{38}(\xi,\eta) - \frac{\mu}{2}\Psi_{39}(\xi,\eta) + \mu le^{-l(\xi+\mu\eta)}\Psi_{40}(\xi,\eta), \quad (11)$$

$$P_2(\xi,\eta) = -\rho_1 dU_2(\xi,\eta). \quad (12)$$

where

$$\Psi_{35}(\xi,\eta)=\left[U_1\left(\frac{(\xi-\mu\eta)+(\xi_a+\mu h)}{2}-\frac{(\xi-\mu\eta)+(\xi_a+\mu h)}{2\mu}\right)-\right.$$
$$\left.-\frac{1}{\mu}V_1\left(\frac{(\xi-\mu\eta)+(\xi_a+\mu h)}{2}-\frac{(\xi-\mu\eta)+(\xi_a+\mu\eta)}{2\mu}\right)\right],$$

$$\Psi_{36}(\xi,\eta)=\left[U_1\left(\frac{(\xi+\mu\eta)+(\xi_a-\mu h)}{2}-\frac{(\xi+\mu\eta)+(\xi_a+3\mu h)}{2\mu}\right)-\right.$$
$$\left.-\frac{1}{\mu}V_1\left(\frac{(\xi+\mu\eta)+(\xi_a-\mu h)}{2}-\frac{(\xi+\mu\eta)+(\xi_a+3\mu h)}{2\mu}\right)\right],$$

$$\Psi_{37}(\xi,\eta)=\int_{z_{02}}^{\xi+\mu\eta}e^{lz}\left[U_1\left(\frac{z+(\xi_a-\mu h)}{2}-\frac{z+(\xi_a+3\mu h)}{2\mu}\right)-\frac{1}{\mu}V_1\left(\frac{z+(\xi_a-\mu h)}{2}-\right.\right.$$
$$\left.\left.-\frac{z+(\xi_a+3\mu h)}{2\mu}\right)\right]dz,$$

$$\Psi_{38}(\xi,\eta)=\left[U_1\left(\frac{(\xi-\mu\eta)+(\xi_a+\mu h)}{2}-\frac{(\xi-\mu\eta)+(\xi_a+\mu h)}{2\mu}\right)-\right.$$
$$\left.-\frac{1}{\mu}V_1\left(\frac{(\xi-\mu\eta)+(\xi_a+\mu h)}{2}-\frac{(\xi-\mu\eta)+(\xi_a+\mu\eta)}{2\mu}\right)\right],$$

$$\Psi_{39}(\xi,\eta)=\left[U_1\left(\frac{(\xi+\mu\eta)+(\xi_a-\mu h)}{2}-\frac{(\xi+\mu\eta)+(\xi_a+3\mu h)}{2\mu}\right)-\right.$$
$$\left.-\frac{1}{\mu}V_1\left(\frac{(\xi+\mu\eta)+(\xi_a-\mu h)}{2}-\frac{(\xi+\mu\eta)+(\xi_a+3\mu h)}{2\mu}\right)\right],$$

$$\Psi_{40}(\xi,\eta)=\int_{z_{02}}^{\xi+\mu\eta}e^{lz}\left[U_1\left(\frac{z+(\xi_a-\mu h)}{2}-\frac{z+(\xi_a+3\mu h)}{2\mu}\right)-\frac{1}{\mu}V_1\left(\frac{z+(\xi_a-\mu h)}{2}-\right.\right.$$
$$\left.\left.-\frac{z+(\xi_a+3\mu h)}{2\mu}\right)\right]dz,$$

Now let's start solving the problem in region 3. For this we have the equation [6,7-10]

$$\mu^2\frac{\partial^2\varphi_3}{\partial\xi^2}-\frac{\partial^2\varphi_3}{\partial\eta^2}=0, \tag{13}$$

and boundary conditions

$$V_3-V_2=-\mu(U_3-U_2) \quad \text{at} \quad \xi-\mu\eta=2\mu h, \tag{14}$$

$$P_3(\xi,0)=f(\xi) \quad \text{at} \quad \eta=0,\ \xi_b\le\xi\le\xi_d. \tag{15}$$

We represent the solution of equation (6.6.6) in the form

$$\varphi_3(\xi,\eta)=f_3(\xi-\mu\eta)+f_4(\xi+\mu\eta). \tag{16}$$

Then, substituting (16) into (14) and (15) to find the required functions $f_3(z)$ and $f_4(z)$ we get the formulas

$$f_3'(z) = -\frac{1}{2\mu}\left[V_2\left(\frac{z}{2}+\mu h, \frac{z-2\mu h}{2\mu}\right)+\mu U_2\left(\frac{z}{2}+\mu h, \quad \frac{z-2\mu h}{2\mu}\right)\right]-\frac{f(z)}{\rho_1 d}, \quad (17)$$

$$f_3'(z) = -\frac{1}{2\mu}\left[V_2\left(\frac{z}{2}+\mu h, \frac{z-2\mu h}{2\mu}\right)+\mu U_2\left(\frac{z}{2}+\mu h, \quad \frac{z-2\mu h}{2\mu}\right)\right]-\frac{f(z)}{\rho_1 d}, \quad (18)$$

So, to determine the components of the velocity and pressure of the medium in region 3, we have the formulas

$$U_3(\xi,\eta) = -\frac{1}{2\mu}\Psi_{41}(\xi,\eta)-\frac{f(\xi-\mu\eta)}{\rho_1 d}+\frac{1}{2\mu}\Psi_{42}(\xi,\eta), \quad (19)$$

$$V_3(\xi,\eta) = \frac{1}{2}\Psi_{43}(\xi,\eta)+\frac{f(\xi-\mu\eta)}{\rho_1 d}+\frac{1}{2}\Psi_{44}(\xi,\eta), \quad (20)$$

$$P_3(\xi,\eta) = -\rho_1 d U_3(\xi,\eta). \quad (21)$$

where

$$\Psi_{41}(\xi,\eta) = \left[V_2\left(\frac{(\xi-\mu\eta)}{2}+\mu h, \frac{(\xi-\mu\eta)-2\mu h}{2\mu}\right)+\right.$$
$$\left.+\mu U_2\left(\frac{(\xi-\mu\eta)}{2}+\mu h, \frac{(\xi-\mu\eta)-2\mu h}{2\mu}\right)\right],$$

$$\Psi_{42}(\xi,\eta) = \left[V_2\left(\frac{(\xi+\mu\eta)}{2}, \frac{(\xi-\mu\eta)-2\mu h}{2\mu}\right)+\mu U_2\left(\frac{(\xi+\mu\eta)}{2}+\mu h, \frac{(\xi+\mu\eta)-2\mu h}{2\mu}\right)\right],$$

$$\Psi_{43}(\xi,\eta) = \left[V_2\left(\frac{(\xi-\mu\eta)}{2}+\mu h, \frac{(\xi-\mu\eta)-2\mu h}{2\mu}\right)+\mu U_2\left(\frac{(\xi-\mu\eta)}{2}+\mu h, \frac{(\xi-\mu\eta)-2\mu h}{2\mu}\right)\right],$$

$$\Psi_{44}(\xi,\eta) = \left[V_2\left(\frac{(\xi+\mu\eta)}{2}+\mu h, \frac{(\xi-\mu\eta)-2\mu h}{2\mu}\right)+\mu U_2\left(\frac{(\xi+\mu\eta)}{2}+\mu h, \frac{(\xi+\mu\eta)-2\mu h}{2\mu}\right)\right],$$

The solution to the problem for the subsequent areas is not given, since it is constructed in a similar way. If the gasket material has a rigid plastic property, i.e. $\sigma = \sigma_s = const,$ then for the solution of the problem in the region of 2 substitutions (3) we have the condition [1-4,10]

$$U_2(\xi,\eta) = -\frac{\sigma_s}{\rho_1 d} \quad \text{at} \quad \eta = h, \quad \xi_a \le \xi \le \xi_c \quad (22)$$

In this case, the unknown function $f_2(z)$, in contrast to (8), is found using the formula

$$f_2'(z) = -f_1'(z - 2\mu h) - \frac{\sigma_s}{\rho_1 d}. \qquad (23)$$

Then the velocity components $U_2(\xi,\eta)$ and $V_2(\xi,\eta)$ in region 2 are represented as

$$U_2(\xi,\eta) = \frac{1}{2}\Psi_{45}(\xi,\eta) - \frac{1}{2}\Psi_{46}(\xi,\eta) - \frac{\sigma_s}{\rho_1 d}, \qquad (24)$$

$$V_2(\xi,\eta) = -\frac{\mu}{2}\Psi_{47}(\xi,\eta) - \frac{\mu}{2}\Psi_{48}(\xi,\eta) - \frac{\sigma_s \mu}{\rho_1 d}. \qquad (25)$$

where

$$\Psi_{45}(\xi,\eta) = \frac{1}{2}\left[\left[U_1\left(\frac{(\xi-\mu\eta)}{2} + \mu h, h - \frac{tg\beta}{2}(\xi-\mu\eta)\right)\right] - tg\beta \cdot V_1\left(\frac{(\xi-\mu\eta)}{2} + \mu h, h - \frac{tg\beta}{2}(\xi-\mu\eta)\right)\right],$$

$$\Psi_{46}(\xi,\eta) = \left[U_1\left(\frac{(\xi+\mu\eta)}{2}, -\frac{tg\beta}{2}((\xi+\mu\eta) - 2\mu h)\right) - tg\beta \cdot V_1\left(\frac{(\xi+\mu\eta)}{2}, -\frac{tg\beta}{2}((\xi+\mu\eta) - 2\mu h)\right)\right],$$

$$\Psi_{47}(\xi,\eta) = \left[\left[U_1\left(\frac{(\xi-\mu\eta)}{2} + \mu h, h - \frac{tg\beta}{2}(\xi-\mu\eta)\right)\right] - V_1 tg\beta \cdot V_1\left(\frac{\xi-\mu\eta}{2} + \mu h, h - \frac{tg\beta}{2}(\xi-\mu\eta)\right)\right],$$

$$\Psi_{48}(\xi,\eta) = \left[U_1\left(\frac{(\xi+\mu\eta)}{2}, -\frac{tg\beta}{2}((\xi+\mu\eta) - 2\mu h)\right) - tg\beta \cdot V_1\left(\frac{(\xi+\mu\eta)}{2}, -\frac{tg\beta}{2}((\xi+\mu\eta) - 2\mu h)\right)\right],$$

In order to study the effect of laying on soil parameters, it is necessary to carry out a series of calculations on a PC for areas 2 and 3.

Conclusion. Mathematical models of wave propagation under the influence of a moving load on a nonlinearly compressible and irreversible unloading soil layer with a base have been built. An analytical solution is obtained for the problem of the propagation of a plastic wave in the case when the relationship between pressure and volumetric deformation during loading and unloading is linear, but different. Based on the analysis of the calculation results, it is shown that if the moving load acting on the boundary has a monotonically decreasing profile, then in the perturbation region, the medium is unloaded and an oblique compression wave is obtained by the load-unloading wave. The pressure of the medium against the background of this wave, depending on the depth, decreases slowly than on the free surface. In the case when the dependence between P and under loading of the medium is assumed to be nonlinear and shock, which corresponds to the propagation of a shock wave in the medium, the pressure in the perturbed region, in comparison with the linear case, is somewhat overestimated.

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