Heat production during deformation of inhomogeneous bodies

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Abstract. A model of a rigid body with internal stresses and is presented, which makes it possible to describe the dissipation of energy when changing from the elastic stage of deformation to the plastic one. The dependence of heat release on the heat-physical properties of the contacting structures is noted.

Keywords: internal stresses, thermal conductivity in inhomogeneous media, contact thermal disturbances

In various technological processes of metal processing, residual stresses (internal, intrinsic [1]) are formed, which usually remain in parts after their manufacture. One of the important characteristics of a deformed solid is the strain tensor [2].

$$\varepsilon_{ij} = \hat{g}_{ij} - \mathring{g}_{ij} \quad (1)$$

which is introduced as a result of comparisons of two states of the body: the data considered by $\hat{g}_{ij}$ and the "initial" $\mathring{g}_{ij}$. There are theories in which the "initial" state is taken to be a state that is not really realized. This is the case when the metal solidifies and as a result of preliminary plastic deformation. Then, we can write down

$$\varepsilon_{ij} = \varepsilon_{ij}' + \varepsilon_{ij}^{*} \quad (2)$$

where ε_{ij}' is expressed in terms of displacement and satisfies the deformation compatibility equations, and ε_{ij}^{*} - is not expressed through displacement and, generally speaking, does not satisfy the conditions of compatibility. The tensor components ε_{ij}^* describe the "initial" deformed state. It is easy to see that the internal metric ε_{ij}^* can be Euclidean only in the absence of internal stresses [3]. Therefore, ε_{ij}^*, describes the incompatibility of deformation and generates internal stresses [3].

The influence of residual (internal) stresses on strength under static and dynamic loads is known [1], the presence of internal stresses is unknown to us. In this paper, it is proposed to consider the features of heat generation during deformation of metals, taking into account internal stresses.

Approximate solution of some problems of heat conduction in contact with bodies with different thermophysical properties.

The formation of an inhomogeneous (granular) structure of metals in various technological processes occurs in different ways. Their occurrence is usually based on irreversible volumetric changes in the material. Therefore, in practice, problems are often encountered associated with the calculation of thermal conductivity in an inhomogeneous medium. The solution of such problems associated with the stepped behavior of the thermal diffusivity depending on heating and cooling [1] is fraught with great difficulties. In this regard, it is advisable to consider approximate methods for solving the heat equation based on the satisfaction of integral relations.

The process of thermal conductivity in a material is described by the Fourier equation:

$$\frac{\partial T}{\partial t} = \frac{\lambda}{c\gamma} \times \frac{\partial^2 T}{\partial \gamma^2}$$

T-temperature, λ-coefficient of thermal conductivity, c- heat capacity of the material, γ- density.

As a boundary condition, we will consider the heat flux on the wall, that is, boundary conditions of the second kind. For a semi-bounded body and a constant heat flux q_λ, going to heat the material, we write approximately the temperature profile in the form of a quadratic parabola.

$$T - T_0 = \frac{q_\lambda}{\lambda} \frac{(\delta_\lambda - y)^2}{2\delta_\lambda} \quad (3)$$

δ_λ- material heating thickness, T_0- the initial temperature of the material, in what follows we will assume $T_0 = const = 0$.

Let us integrate (2) within $0 \le y \le \delta_\lambda$ taking into account the boundary condition:

$$q_\lambda = -\lambda \frac{\partial T}{\partial y} \quad (4)$$

$$\frac{d}{dt}\int_0^{\delta_\lambda} T dy = \frac{q\lambda}{\rho c} \quad (5)$$

We integrate (5) over time and substitute approximation (3) into (5), we obtain:

$$\int_0^{\delta_\lambda} q_\lambda \frac{(\delta_\lambda - y)^2}{2\delta_\lambda} dy = q_\lambda t\alpha \quad (6)$$

Where $\alpha=\frac{\lambda}{\rho C_v}$ – thermal diffusivity coefficient. From (6) we determine the heating thickness:

$$\delta_A = \sqrt{6\alpha t} \quad (7)$$

Thus, the temperature profile is described by the expression:

$$T(y,t) = \frac{q\lambda}{\lambda}\frac{\left(\sqrt{6\alpha t} - y\right)^2}{2\sqrt{6\alpha t}} \quad (8)$$

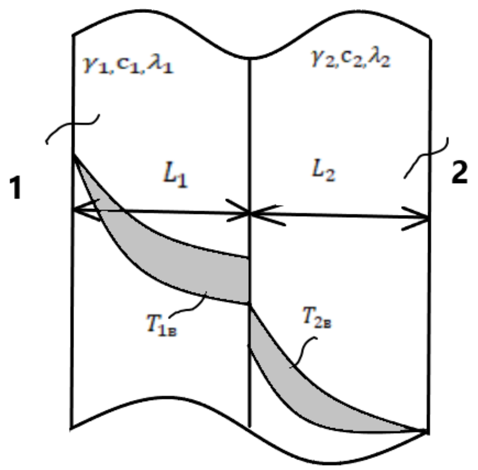
The exact solution to this problem [4]

$$T(o,t) = 2\frac{q\lambda}{\lambda}\sqrt{\frac{\lambda t}{\pi}} \quad (9)$$

The error does not exceed 9%, which is acceptable for applied tasks.

Now let us consider the process of thermal conductivity in the presence of contact between different materials. The process of thermal conductivity in material "1" does not depend on the thermophysical properties of material "2" and vice versa.

Consider the contact of two plates of finite thickness with different thermophysical characteristics (Fig. 1).



(Fig. 1 Temperature profile near contact surfaces)

Let the temperature distribution function in material 1 $(0 \leq y \leq l_1)$ be the same as that of a semi-bounded body. Let us assume that the boundary conditions on the wall change in such a way that a constant flow q_∞ acts on the contact line for a certain time interval Δt.

The origin of q_∞ is explained by the existence of internal stresses δ^* in the material and the coefficient of friction f of the contact planes (Coulomb's Law).

Then the temperature of the contacting surfaces at the end of the time interval Δt will be

$$\Delta T_1 = \frac{q_\lambda}{2\lambda_1}\sqrt{6\alpha_1 \Delta t}$$

$$\Delta T_2 = \frac{q_\infty}{2\lambda_2}\sqrt{6\alpha_2 \Delta t} \quad (10)$$

It follows from (10) that for $\frac{\sqrt{\alpha_1}}{\lambda_1} \neq \frac{\sqrt{\alpha_2}}{\lambda_2}$ (11)

There will be a temperature gap on the contact line.

In reality, of course, there is no temperature gap. Consequently, the assumption about the mutual independence of thermal conductivity upon contact of different materials is not true, since the following conditions must be met on the contact line:

$$T_1 = T_2$$

$$\lambda_1 \frac{\partial T_1}{\partial y} = \lambda_2 \frac{\partial T_2}{\partial y} \quad (12)$$

Then we represent the temperature in the vicinity of the contact line as a sum:

$$T_1 = T_{1\infty} - T_{1B}$$

$$T_2 = T_{2\infty} + T_{2B} \quad (13)$$

$T_{1\infty}$, $T_{2\infty}$ temperatures without taking into account mutual influence, calculated for a semi-infinite space;

T_{1B} and T_{2B} – temperature components due to the mutual influence of materials on thermal conductivity (contact disturbances).

Based on the conditions of the heat balance, it is necessary that the equality is observed: (Fig. 1)

$$-c_1\gamma_1 \int_0^{L_1} T_{1B} dy + c_2\gamma_2 \int_{L_1}^{L_1+L_2} T_{2B} dy = 0 \quad (14)$$

Hence it follows that the temperatures of contact disturbances should have opposite signs (13), and the heat fluxes of contact disturbances q_B are equal and mutually opposite (Fig. 1). As applied to the example (Fig. 1), we have

$$q_B = -\lambda_1 \left(\frac{\partial T_{1B}}{\partial y}\right)_{L_1} = \lambda_2 \left(\frac{\partial T_{2B}}{\partial y}\right)_{L_1} \quad (15)$$

Thus, taking into account the mutual influence of contacting materials is reduced to determining the magnitude of the heat flux of the contact disturbance q_B. From (10), (12), (13) we get:

$$\frac{(q_\infty - q_B)}{2\lambda_1}\sqrt{6\alpha_1 \Delta t} = \frac{(q_\infty + q_B)}{2\lambda_2}\sqrt{6\alpha_2 \Delta t} \quad (16)$$

From (16) we define

$$q_B = q_\infty \frac{1-k}{1+k} \quad (17)$$

$$k = \sqrt{\frac{c_1 \gamma_1 \lambda_1}{c_2 \gamma_2 \lambda_2}} \quad (18)$$

You can name the k-coefficient of thermal activity of the material "2" in relation to the material "1". Relation (17) was obtained with a constant heat flux on the contact line. Since the equation of thermal conductivity is linear, the obtained relation is valid for any law of change on the contact surface.

It follows from (17) that the magnitude of the thermal contact disturbance depends on the ratio of the thermophysical properties of materials (contacting grains with different internal stresses, etc.)

At k=1, $q_B = 0 -$ there are no thermal disturbances, the material is thermophysically homogeneous.

If k<1, then $q_B > 0$, which means that material "2" has a cooling effect on material "1".

At k=0- corresponds to absolute cooling.

If k>1, then $q_B < 0 -$ in this case material "2" turns out to be a heat insulator.

k=∞ corresponds to the absolute heat insulator. The heat flux on the contact line will be zero. There is a complete reflection of the undisturbed heat flux from the line of contact with the absolute heat insulator, and this reflected flux goes to heating material "1". At $1 < k < \infty$ only partial reflection takes place, that is, a part of the thermal energy arriving at the contact surface goes to heating material "2" by thermal conduction.

References

1. I.A. Kunin. Theory of elastic media with microstructure.
2. L.I. Sedov. Continuum mechanics.
3. I.A. Birger, Residual stresses
4. A.V. Lyskov. Heat conduction theory